SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2015
HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK # 2

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 90 Minutes
- Write using black or blue pen. Black pen is preferred.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- In Questions 8 - 10, show **ALL** relevant mathematical reasoning and/or calculations.
- Marks may **NOT** be awarded for untidy or badly arranged work.
- Answer in simplest **EXACT** form unless otherwise instructed.

Total marks - 52

Section I Pages 2–4 (**7 marks)**

- Attempt Questions 1–7 on the Multiple Choice answer sheet provided.
- Allow about 10 minutes for this section.

Section II Pages 5–15 (**45 marks)**

- Attempt Questions 8–10.
- Start a new answer booklet for each question.
- Allow about 1 hours and 20 minutes for this section.

Examiner: V. Likourezos

This is an assessment task only and does not necessarily reflect the content of the Higher School Certificate.

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Section I

7 marks
Attempt Questions 1–7
Allow about 10 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–7.

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1 Seven people are to be seated around a circular table. If two particular people must be seated together, how many seating arrangements are possible?

(A) $7!$

(B) $5!\times 2$

(C) $6!\times 2$

(D) $6!$

2 It is known that an approximate root to the curve $y=e^x-3x^2$ is $x=3.8$. Using Newtons Method of Approximation with one application what is an approximation of the root?

Round your answer correct to 2 decimal places.

(A) $x=3.74$

(B) $x=4.22$

(C) $x=-12.06$

(D) $x=3.70$

3 What is the amplitude and period of the graph $y=2\pi\cos\left(2x+\dfrac{\pi}{3}\right)$?

(A) amplitude $=2$ and period $=\pi$

(B) amplitude $=2\pi$ and period $=\pi$

(C) amplitude $=2\pi$ and period $=2$

(D) amplitude $=2$ and period $=2$

4 Four female and four male athletes are arranged in a row for the presentation of prizes. In how many ways can this be done if the males and females must alternate?

(A) $4!\times 4!$

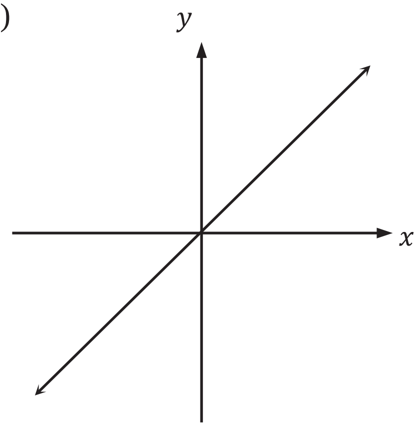
(B) $2\times 4!\times 4!$

(C) $4!\times 5!$

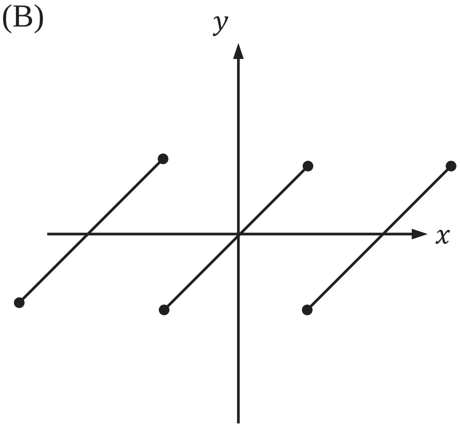
(D) $2\times 4!\times 5!$

5 Which diagram best represents the function $y=\sin^{-1}\left(\sin x\right)$?

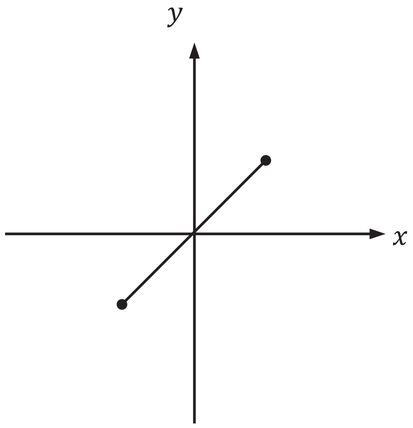
(A)



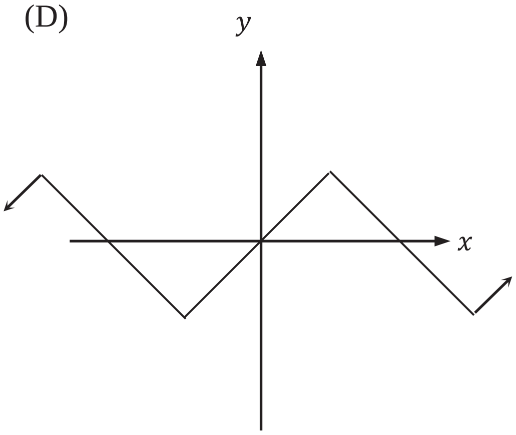
(B)



(C)



(D)



6 What is the value of $\dfrac{(n+r+1)!}{(n+r-1)!}$ in simplest form?

(A) $n(n+r)$

(B) $n+r+1$

(C) $n(n+r-1)$

(D) $(n+r)(n+r+1)$

7 What is the derivative of $y = 3^{3x+1}$?

(A) $\dfrac{dy}{dx} = (3x+1)\times 3^{3x}$

(B) $\dfrac{dy}{dx} = 3\times 3^{3x+1}\ln 3$

(C) $\dfrac{dy}{dx} = 3^{3x+1}\ln 3$

(D) $\dfrac{dy}{dx} = 3\times 3^{3x+2}\ln 3$

End of Section I

Section II

45 marks
Attempt Questions 8 –10
Allow about 1 hour and 20 minutes for this section.

Answer each question in a new writing booklet. Extra writing booklets are available.

In Questions 8–10, your responses should include ALL relevant mathematical reasoning and/or calculations.

Question 8 (15 marks) Start a new writing booklet for each question.

(a) Using the table of standard integrals, find **2**

$$\int 3\sec\left(\frac{1}{3}x\right)\tan\left(\frac{1}{3}x\right)dx$$

(b) Differentiate $y = x\cos\left(x^2+1\right)$ with respect to x. **2**

(c) Find $\displaystyle\int \frac{\cos x}{1+\sin x}\, dx$ **1**

(d) (i) Let $f(x) = \ln\left(x+\sqrt{x^2-1}\right)$, what is the domain of $f(x)$? **1**

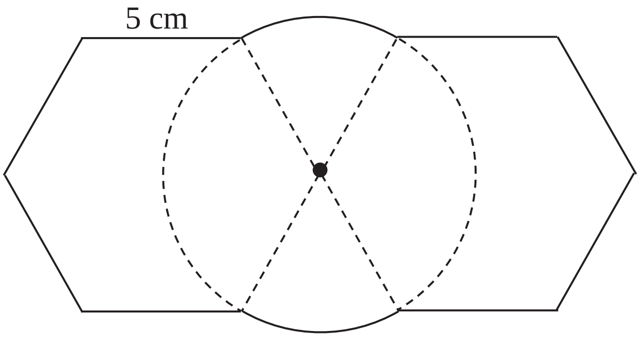
(ii) Show that the derivative $f'(x) = \dfrac{1}{\sqrt{x^2-1}}$ **2**

(e) (i) Find $\dfrac{d}{dx}\left(x^2e^{x^2}\right)$ **2**

(ii) Hence, evaluate $\displaystyle\int_0^1 x^3e^{x^2}\, dx$ **3**

Question 8 continued

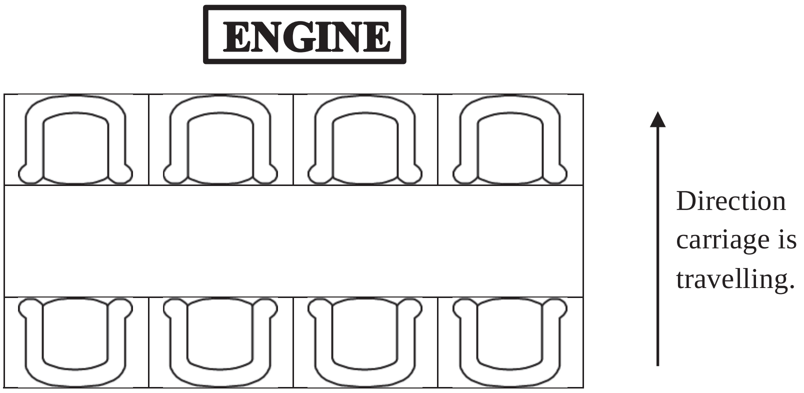
(f) The diagram above shows two regular hexagons joined to both sides of a circle. The corner of the hexagons meets the circle at its centre. The side length of the hexagon is 5 cm. Find the exact perimeter of the resulting shape. **2**



End of Question 8

Question 9 (15 marks) Start a new writing booklet for each question.

(a) Five people enter a railway carriage in which there are 8 empty seats. Below is a diagram showing the layout of the seats.



In how many ways can they take places if:

(i) Any person can sit in any seat? **1**

(ii) If one of the passengers, Chloe, sits in a corner? **1**

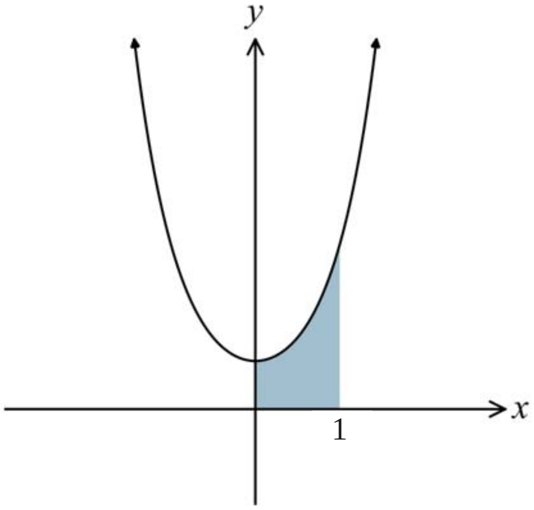
(iii) Chloe sits with her back to the engine while her friend, Vicki, sits facing the engine? **1**

(b) (i) Show that $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \sin^2 2x \, dx = \frac{1}{8}(\pi + 2)$ **3**

(ii) Hence or otherwise find the volume of the solid of revolution formed when the curve $y = 1 + \sin 2x$ is rotated about the x-axis between $x = \frac{\pi}{8}$ and $x = \frac{3\pi}{8}$. Give your answer correct to 3 significant figures. **3**

Question 9 continued

(c) In physics and geometry, a catenary is the curve that an idealised hanging chain or cable assumes under its own weight when supported only at its ends. The diagram below shows the region bounded by the catenary, the x-axis, the y-axis and the line $x = 1$.



The equation of the catenary is given by $y = \dfrac{e^x + e^{-x}}{2}$.

(i) Find the exact area of the shaded region. **2**

(ii) Using Simpson's rule with three function values, find an approximation to the shaded area correct to 2 decimal places. **2**

(iii) Hence deduce that $e \approx 2.7$ correct to 1 decimal place. **2**

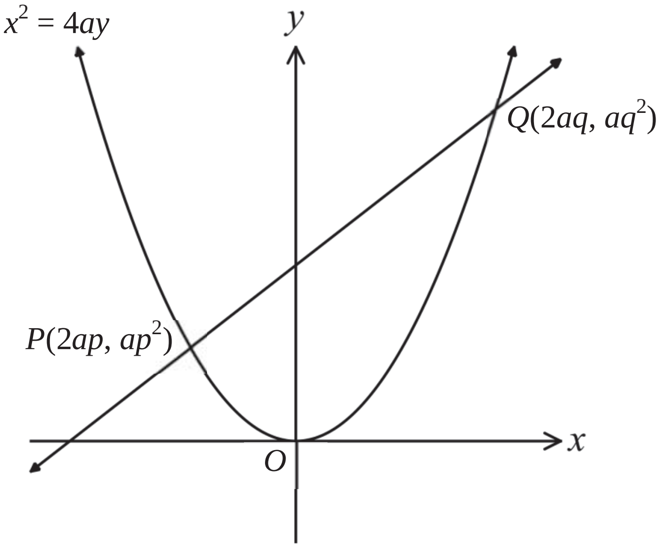
End of Question 9

Question 10 (15 marks) Start a new writing booklet for each question.

(a) Evaluate $\lim_{x\to 0}\dfrac{\sin 2x}{5x}$ **1**

(b) Prove by mathematical induction $7^n + 3n \times 7^n - 1$ is divisible by 9 for all positive integers *n*. **3**

(c) The points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ lie on the parabola $x^2 = 4ay$.



The variable chord *PQ* is such that it is always parallel to the line $y = 2x$.

(i) Show that $p + q = 4$. **1**

(ii) Show that the equation of the normal at *P* is $x + py = 2ap + ap^3$. **2**

(iii) The normals at *P* and *Q* intersect at $R\left(-apq\left(p+q\right),\ 2a + a\left(p^2 + pq + q^2\right)\right)$. **2**
Find the locus of *R*. (Ignore any restrictions).

(iv) Find the domain of the locus of *R*. **1**

Question 10 continued

(d) Consider the pair of simultaneous equations

$$y = \sin x \cos x$$
$$y = mx$$

(i) Suppose m is positive. By sketching, or otherwise, find any restrictions on m so that it will have a unique simultaneous solution. **2**

(ii) Suppose m is negative. Show that the pair of equations have a unique simultaneous solution if $m < \cos\theta$, where θ satisfies the equation $\tan\theta = \theta$ for $\pi < \theta < \frac{3\pi}{2}$. **3**

End of paper

End of Exam

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad n \neq 0; \ \text{if } n < 0.$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right),$$

Note: $\ln x = \log_e x \qquad x > 0$

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2015

HSC Task #2

Mathematics Extension 1

Suggested Solutions
&
Markers' Comments

QUESTION	Marker
1 – 7	–
8	PB
9	BK
10	BD

Multiple Choice Answers

1.	B
2.	A
3.	B
4.	B
5.	D
6.	D
7.	B

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(C) amplitude $=2\pi$ and period $=2$
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[✓ (B)] $2\times4!\times4!$

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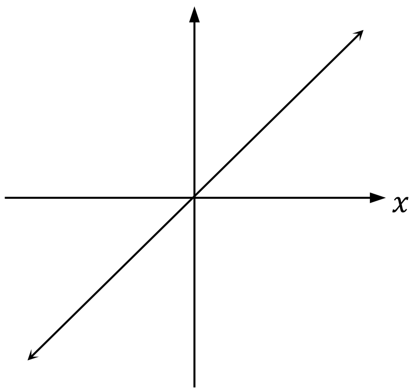
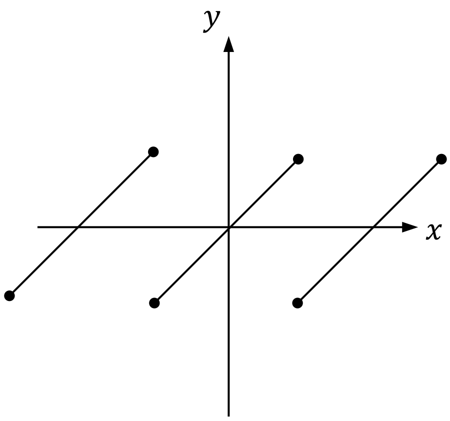
5 Which diagram best represents the function $y=\sin^{-1}(\sin x)$?

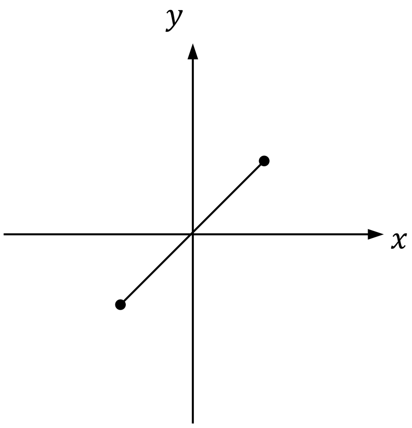
(A)



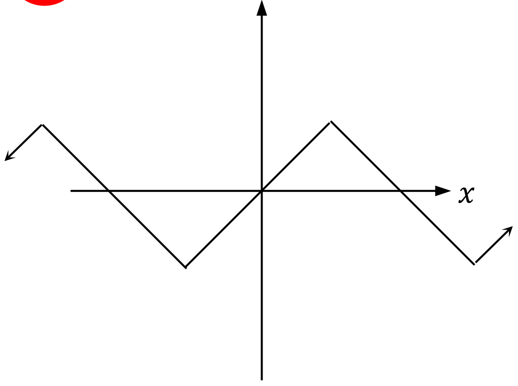
(B)



(C)



[✓ (D)]



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[✓ (D)] $(n+r)(n+r+1)$

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COMMENTS & SOLUTIONS FOR QUESTION 8.

(X1)

a) $9 \sec \frac{x}{3}$

COMMENT: Well done. Very few were unable to answer correctly.

b) $y' = \cos(x^2+1) - 2x^2 \sin(x^2+1)$

COMMENT: Well done.

c) $\int \frac{\cos x \, dx}{1+\sin x} = \ln(1+\sin x) + c.$

COMMENT: Very few students failed to get full marks.

d) (i) $x \geq 1.$

COMMENT: The common error was to consider $x^2 - 1 \geq 0$

i.e. $x \leq -1, x \geq 1.$

Clearly if $x \leq -1$ then $x + \sqrt{x^2-1} < 0$ & hence $\ln\{x + \sqrt{x^2-1})$ is undefined

(ii) If $f(x) = \ln\left(x + \sqrt{x^2-1}\right)$

$$f'(x) = \frac{1 + \frac{1}{2}(x^2-1)^{-\frac{1}{2}} \times 2x}{x + \sqrt{x^2-1}}$$

$$= \frac{1 + \frac{x}{\sqrt{x^2-1.}}}{x + \sqrt{x^2-1}}$$

$$= \frac{\sqrt{x^2-1} + x}{\sqrt{x^2-1}} \times \frac{1}{x + \sqrt{x^2-1.}}$$

$$= \frac{1}{\sqrt{x^2-1}}.$$

COMMENT:

A significant number of students used the Standard Integrals to find the answer.

Well done.

(e) (i) Find $\frac{d}{dx}\left(x^2 e^{x^2}\right) = x^2 \times 2x e^{x^2} + 2x e^{x^2}$

$$= 2x e^{x^2}\left(x^2+1\right)$$

(ii) Hence $\int_0^1 2x e^{x^2}(x^2+1)dx = \left[x^2 e^{x^2}\right]_0^1$

$$= e$$

$$\therefore 2\int_0^1 x^3 e^{x^2} dx + \int_0^1 2x e^{x^2} dx = e$$

$$\text{ie. } 2\int_0^1 x^3 e^{x^2}\,dx + \left[e^{x^2}\right]_0^1 = e.$$

$$2\int_0^1 x^3 e^{x^2}\,dx + (e-1) = e$$

$$2\int_0^1 x^3 e^{x^2}\,dx = 1$$

$$\int_0^1 x^3 e^{x^2}\,dx = \frac{1}{2}.$$

COMMENT. Part (i) was well done.

Some struggled with (ii).

(d) Perimeter $= 8 \times 5$ cm $+ \frac{1}{3}$ of $2\pi \times 5$.

$= \left(40 + \frac{10\pi}{3}\right)$ cm.

COMMENT. most were able to gain full marks.

GENERAL COMMENT Very few marks below 10 out of 15. a significant number attained full marks.

Q.9. Ext 1.

(a) (i) ${}^8P_5 = 6720$ ways ✓

(ii) $4 \times {}^7P_4 = 3360$ ways ✓

(iii) $4 \times 4 \times {}^6P_3 = 1920$ ways ✓

③

Q9(a) For the most part students either did all correctly aor all incorrectly. Errors included using combinations or using n! instead of ${}^nP_r$

(b) $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \sin^2 2x \, dx$ Now $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$$= \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{2}(1-\cos 4x)\,dx \quad ✓$$

$$= \frac{1}{2}\left[x - \frac{\sin 4x}{4}\right]_{\frac{\pi}{8}}^{\frac{3\pi}{8}}$$

$$= \frac{1}{2}\left[\left(\frac{3\pi}{8} - \frac{\sin\frac{3\pi}{2}}{4}\right) - \left(\frac{\pi}{8} - \frac{\sin\frac{\pi}{2}}{4}\right)\right]$$

$$= \frac{1}{2}\left(\frac{3\pi}{8} + \frac{1}{4} - \frac{\pi}{8} + \frac{1}{4}\right) \quad ✓$$

$$= \frac{1}{2}\left[\frac{\pi}{4} + \frac{1}{2}\right]$$

$$= \frac{1}{8}(\pi + 2) \quad \# \quad ✓$$

③

(b)(i) Mostly correct. Some errors with signs of trig functions.

(b) (ii) $V = \int_{\frac{\pi}{8}}^{8} \pi (1 + \sin 2x)^2 \, dx$

$= \pi \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} (1 + 2\sin 2x + \sin^2 2x)\, dx$ ✓

$= \pi \left[x - \frac{\cancel{2}\cos 2x}{\cancel{2}} \right]_{\frac{\pi}{8}}^{\frac{3\pi}{8}} + \pi \times \underbrace{\frac{1}{8}(\pi + 2)}_{\text{from(i)}}$

$= \pi \left[\frac{3\pi}{8} - \cos\frac{3\pi}{4} - \left(\frac{\pi}{8} - \cos\frac{\pi}{4}\right)\right] + \frac{\pi}{8}(\pi + 2)$

$= \pi \left(\frac{\pi}{4} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) + \frac{\pi}{8}(\pi + 2)$ ✓

$= \frac{\pi^2}{4} + \frac{\pi^2}{8} + \pi\left(\frac{2}{\sqrt{2}} + \frac{1}{4}\right)$

$= \frac{3\pi^2}{8} + \pi\left(\frac{\cancel{2}\sqrt{2}}{\cancel{2}} + \frac{1}{4}\right)$

$= \frac{3\pi^2}{8} + \pi\left(\sqrt{2} + \frac{1}{4}\right)$

$= 3.70110 + 5.22828$

$= 8.9293$

$= 8.93$ to 3sf ✓ ③

(ii) Again mostly correct. Errors tended to be arithmetic.

(c) $y = \frac{e^x + e^{-x}}{2}$

i) Area $= \frac{1}{2}\int_0^1 (e^x + e^{-x})\,dx$ ✓

$= \frac{1}{2}\left[e^x - e^{-x}\right]_0^1$

$= \frac{1}{2}\left[(e - e^{-1}) - (1-1)\right]$ ②

$= \frac{1}{2}\left(e - \frac{1}{e}\right)$ units^2. ✓

(ii)

x	0	0.5	1
y	1	1.1276	1.543

Area $= \frac{w}{3}\left[\text{ends} + 4\,\text{odds}\right]$

$= \frac{0.5}{3}\left[1 + 1.543 + 4(1.1276)\right]$ ✓ ②

$= \underline{1.175580\, u^2}$. ✓

(iii) $\therefore \frac{1}{2}\left(e - \frac{1}{e}\right) = 1.175580\ \frac{1}{2}$

$e - \frac{1}{e} = 2.351160212$ ②

$e^2 - 2.351160212e - 1 = 0$ ✓

$e = \frac{2.351160212 \pm \sqrt{2.351160212^2 + 4}}{2}$ ✓

$\underline{e = 2.71894}$ (reject negative answer since $e > 0$)

(c) (i) Done well.

(ii) Done well

(iii) Most equated (i) and (ii) to get equation. Most errors involved using the value of e from the calculator in order to solve the equation to find e.

Q10 Y12 Ext 1 Task 2 2015

The mean score for this question was 7.15/15

(a) Most students were able to receive full marks.

10(a) $\lim_{x \to 0} \frac{\sin 2x}{5x}$

$= \lim_{2x \to 0} \frac{2}{5} \frac{\sin 2x}{2x}$

$= \frac{2}{5} \cdot 1$

$= \frac{2}{5}$

①

0	0.5	1	Av
15	41	107	0.78

(b) Most students found this Mathematical Induction question difficult. Identifying an appropriate way to use the *S*(*k*) expression was difficult.

(b) $S(n) \equiv 7^n + 3n \times 7^n - 1 = 9M_n$, M_n an integer.

i.e $7^n(3n+1) - 1 = 9M_n$

Step 1: show $S(1)$ is true

$LHS = 7^1(3 \times 1 + 1) - 1$

$= 7 \times 4 - 1$

$= 27$

$= 9 \times 3$

$= 9M_1$, M_1 an integer

Step 2: Assume $S(k)$ is true

i.e $7^k(3k+1) - 1 = 9M_k$, M_k an integer

Show $S(k+1)$ is true

i.e $7^{k+1}(3k+4) - 1 = 9M_{k+1}$, M_{k+1} an integer

$LHS = (3k+1).7^k.7 + 3.7^{k+1} - 1$

$= 7((3k+1).7^k - 1) + 3.7^{k+1} + 6$

$= 7.9M_k + 3.7.7^k + 6$

$= 7.9M_k + 3.7(9M_k - 3k.7^k + 1) + 6$

$= 7.9M_k + 21.9M_k - 7.9k.7^k + 21 + 6$

$= 9(7M_k + 21M_k - 7k.7^k + 3)$

$= 9M_{k+1}$, M_{k+1} an integer

as the set of integers is closed under +, −, ×.

∴ If $S(k)$ is true, $S(k+1)$ is true

③

$S(1)$ is true, therefore by the process of Mathematical Induction $S(n)$ is true for all integral $n \geq 1$.

Other (better) uses of *S*(*k*) were:

$-1 = 9M_k - (3k+1).7^k$

$7^{k+1} + (3k+3)7^{k+1} - 1$

$= (3k+4).7^{k+1} + 9M_k - (3k+1).7^k$

$= 9M_k + 7^k(21k + 28 - 3k - 1)$

$= 9M_k + 7^k(18k + 27)$

$= 9(M_k + 7^k(2k+3))$

and

$7^{k+1} + (3k+3).7^{k+1} - 1$

$= 28.7^k + 21k.7^k - 1$

$= 7^k + 3k.7^k - 1 + 27.7^k + 18k.7^k$

$= 9M_k + 27.7^k + 18k.7^k$

$= 9(M_k + 3.7^k + 2k.7^k)$

0	0.5	1	1.5	2	2.5	3	Av
1	0	45	75	4	4	34	1.70

(c) (i) Most students were able to receive full marks.

(c) (i) $m_{PQ} = \frac{aq^2 - ap^2}{2aq - 2ap}$

$= \frac{a(q-p)(q+p)}{2a(q-p)}$

$= \frac{p+q}{2}$

As PQ ∥ $y = 2x$

$\frac{p+q}{2} = 2$

$\therefore p+q = 4.$

①

0	0.5	1	Av
21	4	138	0.86

(c) (ii) This part was done well.

(ii) $y' = \frac{2x}{4a}$

At P, $y' = \frac{2.2ap}{4a}$

$= p.$

∴ Eqn of normal is

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

$\therefore py - ap^3 = -x + 2ap$

$\therefore x + py = 2ap + ap^3.$

②

0	0.5	1	1.5	2	Av
5	0	1	0	157	1.93

(c) (iii) Students who remembered that $p + q = 4$ usually did well in this part.

(iii) Eqn of normal at Q is

$x + qy = 2aq + aq^3$

$x = -apq(p+q)$

$y = 2a+a(p^2+pq+q^2)$

$x = -apq \cdot 4$

$\therefore pq = -\frac{x}{4a}.$

$y = 2a + a[(p+q)^2 - pq]$

$= 2a + a[16 - pq]$

$= 2a + a[16 + \frac{x}{4a}]$

$= 2a + 16a + \frac{x}{4}$

$= 18a + \frac{x}{4}$

②

$\therefore 4y = 72a + x$

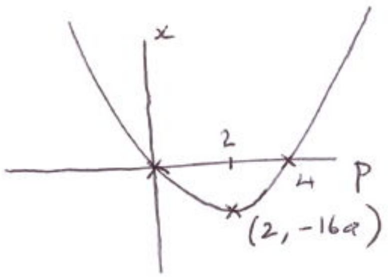
$4y - x = 72a$ is the locus of R

0	0.5	1	1.5	2	Av
39	11	19	15	79	1.26

(c) (iv) No students received marks in this part. Again it was useful to remember that $p + q = 4$.

(iv) $x = -4apq$

$= -4ap(4-p)$



PQ has gradient 2

$p < q$

Tangent occurs when $p = 2 = q$

$\therefore p < 2$ (and $q > 2$)

①

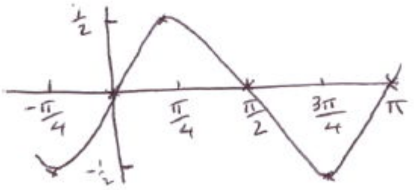
$\therefore x > -16a$

0	0.5	1	Av
163	0	0	0

(d) (i) A number of students realised that the tangent at the origin was important in this question but did not then deduce that *m* = 1 was the lowest value of *m* that would give the required result.

(d) $y = \sin x \cos x = \frac{1}{2} \sin 2x$

$y = mx$



A single root is possible if

$m \geq$ gradient of tangent at O.

$y' = \frac{1}{2} \cos 2x . 2$

$= \cos 2x$

②

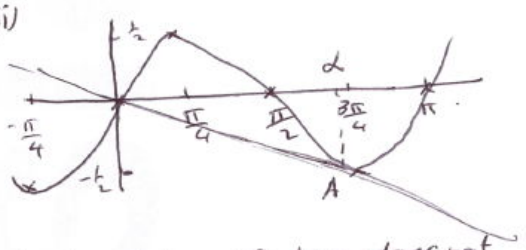
At $x=0$, $y' = 1$

∴ For single solution $m \geq 1$.

0	0.5	1	1.5	2	Av
61	17	26	5	24	0.55

(d) (ii) This part was poorly done; possibly students had run out of time at this stage of the paper.

(ii)



Single solution if line does not touch cut curve (other than at O.

$y' = \cos 2x$

Let A be $(\alpha, \frac{1}{2} \sin 2\alpha)$

∴ Eqn of OA is

$y - \frac{1}{2} \sin 2\alpha = \cos 2\alpha (x - \alpha)$

As the line passes through the origin

$-\frac{1}{2} \sin 2\alpha = -\alpha \cos 2\alpha$

∴ $\sin 2\alpha = 2\alpha \cos 2\alpha$

∴ $2\alpha = \tan 2\alpha$ $\quad \frac{\pi}{2} < \alpha < \frac{3\pi}{4}$

∴ $\pi < 2\alpha < \frac{3\pi}{2}$

OR $\theta = \tan \theta$ $\quad \pi < \theta < \frac{3\pi}{2}$

when $\theta = 2\alpha$

Gradient of OA is $\cos 2\alpha = \cos \theta$

∴ If m is negative for single solution, $m < \cos \theta$

③

0	0.5	1	1.5	2	2.5	3	Av
144	1	14	0	2	1	1	0.15